Border Games: A Game Theoretic Model of Illegal Immigration

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Abstract

We consider a game theoretic model of illegal immigration with an aim to emphasize the strategic interaction between the different forces affecting the issue. Specifically, we analyze the strategic interaction between firms in destination country, native labor, immigrant labor and elected officials in destination country. We show that the impact of regularity actions is dampened because strategic interaction between the players will tend to cancel any unilateral changes. We also study the effect of uncertainty in the labor market on migration issues and analyze the two cases where the policy makers have to make their decisions before (ex-ante) or after (ex-post) the market state is realized.

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1. Introduction

The United States Congress is currently debating changes in the immigration laws and practices for entry into the United States. The proposals discussed would, among a number of changes, grant a path to citizenship for those already in the nation without the proper paperwork as well as change the criterion for legal migration and the number of migrants allowed. Mexico will be affected more than any other nation as approximately 150,000 Mexicans have legally migrated to the United States each year since 2000 [?]. These numbers are supplemented by a large number of undocumented [1] immigrants with some estimates showing as many undocumented migrants from Mexico as legal ones. By a large margin, more migrants enter the United States than from any other nation.3 We use a game theoretic approach to model the immigration of lower skilled Mexican migrants into the United States and the accompanying political process. Our model predicts some somewhat surprising results such as an increase in border enforcement resulting in fewer successful crossing but a higher success rate for those that do attempt to cross. Earlier studies, such as Amegashie [?] and Epstein and Nitzan [?] have concentrated on models of how immigration quotas are set. Karlson and Katz [?] model border patrols, apprehension efforts and amnesty as different facets of the same policy. We contend that in the case of

[1] We use the term undocumented rather than illegal to describe those immigrants who are in the United States without the proper papers.

the United States and Mexico, such legal quotas are only part of the story, with the level of undocumented immigration in some years perhaps higher than that of legal immigrants. Further, the institutions germane to the United States and Mexico, as well as the danger inherent in the border crossing lead to us to a model encompassing many of the unique aspects of this situation. This paper, however, draws on the body of literature such as [?] and [?], to develop a game theoretic model of illegal immigration that yields insights as policies and exogenous factors change.

We developed our theoretical model drawing on Mexico and the United States as the former is the source of majority illegal immigrants to the latter. While the institutions germane to the United States and Mexico, as well as the danger inherent in the US border crossing provided inspiration for our work, the resulting stylized theoretical model of illegal immigration is applicable to many other labor markets across the world.

We assume that there is strong demand for unskilled labor in the United States and a much higher equilibrium wage for this type of labor in the United States than Mexico.4 Although Mexican workers have an incentive to migrate, there are limits on the level of legal migration. Undocumented migration, however, is possible. Our model assumes two countries the destination country (DC) and source country (SC). We assume there is a strong demand for unskilled labor in the destination country and an equilibrium wage much higher than in source country. Source country workers have an incentive to migrate, but there are limits on the level of legal migration. Undocumented migration is, however, possible.

We assume the following players:

- Destination country (DC) firms trying to increase migration both legal and illegal to keep wages low
- Native labor trying to decrease migration to keep wages high
- Source country (SC) families who allocate their resources between a lower paying job at home and a higher paying job, with a risky border crossing, in the destination country
- Elected officials in the destination country who want to get re-elected and will balance the desires of firms, with lobbying funds, and native labor who desire higher wages.

DC firms are attempting to maximize profits which are directly related to the supply of labor and inversely related to the wage rate. The firms will lobby and make political contributions to increase the level of immigrant labor. The government can respond by either increasing the quota of legal immigrants or adjusting the resources put into patrolling the border and reducing illegal immigration. SC families will balance the increased higher wages available in the destination country with both the probability of being apprehended and returned to their native land as well as the risk of injury or death from crossing border. SC families faces a trade off between a risk-free asset of not immigrating and a risky, dangerous but potentially rewarding attempt to migrate. After developing the model we examine the impact of exogenous changes as well as uncertainty.

2. Model

We assume there are two countries; DC (Destination Country) and SC (Source Country). There is strong demand for unskilled labor in DC and the equilibrium wage for these jobs is much higher thus creating an incentive for workers in SC to migrate there. There is no legal immigration program for unskilled workers, so the only option is to illegal migration across the border [2].

We use a game theoretic model where the immigration game is based on the conflicting interests of four players:

SC Families

We assume families living in the source country send a proportion θ of their family's labor force to attempt a border crossing and relocation to DC. The workers trying to cross the border are faced with three outcomes: successfully making to DC and finding a job with a probability p_s, detected and sent back with probability p_b or dying or suffering an injury on the journey with probability p_k where:

$$p_s + p_b + p_k = 1 \tag{1}$$

The expected utility of the SC family is given by:

$$u_{SF} = p_s v_S((1-\theta)w_S + \theta w_D) + p_b v_S((1-\theta)w_S + \theta\rho w_S) + p_k v_S((1-\theta)w_S) \tag{2}$$

where w_S and w_D denote the equilibrium wage in the unskilled labor market in source and destination country respectively.[3] The families choose θ to maximize their utilities. The discount factor ρ captures the lost wages due to unsuccessful attempts crossing the border[4]. Here we adopt a stylized one period model for immigration and assume all migrating members of one family to travel together and the outcome of the border crossing is the same for all. The model captures important characteristics of the border crossing scenario by modeling it as a decision problem of allocating resources between a risky asset (attempting a border crossing with three potential outcomes $\{w_D, \rho w_S, 0\}$) and a risk-free asset (staying and working in SC with a deterministic outcome of $\{w_S\}$).

Firms and Workers in DC

The firms are interested in the availability of a pool of workers for unskilled work. We assume

[2] The United States sets immigration levels with the highest priority being to reunite families. There are some spaces reserved for certain high skill fields facing labor shortages (H1B visas), a limited number of people admitted under a lottery and some refugees granted asylum. There are no spots for unskilled workers, per se. [**?**]

[3] [**?**] analyzing data from the Mexican Migration Project, found that the death rate from suffocation, heat exhaustion, exposure and unknown causes to range between 2 and 6 deaths per 100,000 undocumented migrant crossing from Mexican into the United States in the years 1986-1998. Injury and other health issues can arise as well. Although these are not explicitly placed in the model as costs, if we add some constant to be subtracted from the utility function, the results will not change.

[4] Migrants may be apprehended and returned across the border. In light of the efforts made to get to the border area, these people will try again, usually the next day. The succession of attempts, resulting in either a success, returning home or death and injury can be viewed as a single attempt for our purposes.

the firms can directly observe the number of immigrant workers Q_I. Q_I is the number of illegal immigrants who successfully enter the destination country. The firms are interested in supporting a candidate for elected office who will permit a high number of immigrant workers. The native workers in the destination country are only interested in their average wage rate w_D in this market and do not observe directly the number of immigrant workers. They base their support based on their disposable income level which is proportional to their wage.

Elected Official

The elected official is trying to maximize his chances of re-election. He is trying to balance the interests of the firms and corporations lobbying for higher level of migration. [5] and the welfare of the native workers who are interested in higher wages. We assume the elected official's utility function is of the quasi-linear form.

$$u_{EO} = v_N(w_D) + \alpha Q_I \tag{3}$$

The parameter α weighs the interest of the firms against the welfare of the native workers resulting from changes in the number of immigrants. The elected official shapes the border security policy and determines probability p_s of successfully crossing the border.[6] The elected official can affect the probability of success by changing enforcement levels, i.e. the number of Border Patrol officers or capital such as walls and surveillance equipment.

The workers in the source country and the elected official choose θ and p_s independently without observing each other's choice in a simultaneous move game. Given a particular choice of θ and p_s, we assume the pool of workers in the source country is M, then the number of successful immigrants Q_I is given by:

$$Q_I = M\theta p_s \tag{4}$$

the equilibrium wage in the destination country is given by the demand for labor:

$$w_D = D^{-1}(Q_I + Q_N) = w(Q_I + Q_N). \tag{5}$$

where the D and w denote the demand and inverse demand for labor (we assume $D' < 0, w' < 0$) and Q_N is the number of native workers in DC. The parameters $M, w_S, Q_N, \rho, p_k, \alpha$ are assumed to be exogenously specified. We can divide these parameters in three classes: (i) parameters entering the utility function of the elected official and the labor demand $\{\alpha, M, Q_N\}$ (ii) parameters entering the utility function of only the SC Families $\{w_S, \rho, p_k\}$.

Below we focus on the strategic interaction between the elected official and the SC

[5]Gordon H. Hanson said employers feel very strongly about maintaining access to immigrant workers, and exert political pressure to prevent enforcement from being effective. For example, when the INS conducted raids to apprehend illegal migrants picking onions in Georgia, members of that state's Congressional delegation criticized the agency for the harm caused to farmers [**?**].

[6]For the case of US and Mexico, [**?**] found the actual value of p_s, the probability of successfully crossing the border to be about .67. The figure rose during the early 1990s as Border Patrol officers devoted more time and resources to drug interdiction rather than immigration control.

workers. The interests of the firms and workers in DC is implicit in the utility function of the elected official who is trying to appeal to both firms and native workers in the destination country.

3. Simultaneous Move Game

In this section we consider a simultaneous move game between the elected official and SC families. The elected official chooses p_s and the SC workers, θ without directly observing each other's action.

3.1. Best Response Functions

In the following we first derive the best response function for each player and solve these simultaneously to arrive at the Nash Equilibrium.

SC Families: We recall that the expected utility of a SC family is given by:

$$u_{SF}(p_s, \theta, p_k, w_D, w_S) = p_s v_S(1-\theta)w_S + \theta w_D) + p_b v_S((1-\theta)w_S + \theta\rho w_S) + p_k v_S((1-\theta)w_S) \quad (6)$$

Note that the SC families do not consider the effect of their choice of θ, the proportion of the family to send to the DC, on the DC wage level. We assume that immigrant workers originate from a large number of identical families, each solving the same decision problem given above. The families are faced with a problem of investment choice between a risky asset (attempted immigration) with three potential returns $(w_D, \rho W_S, 0)$ and a risk free asset (remaining in the source country) with the deterministic return of (w_S). Assuming an interior solution $\theta > 0$ the optimal portfolio allocation is given by the first order condition:

$$\begin{aligned} \frac{\partial u_{SF}}{\partial \theta} &= p_s v'((1-\theta)w_S + \theta w_D)(w_S - w_D) - (1 - p_b - p_k)v'((1-\theta)w_S + \theta\rho w_S)(1-\rho)w_S \\ &\quad -p_k v((1-\theta)w_S)w_S = 0 \end{aligned} \quad (7)$$

Equation 7 implicitly defines the best response function f_{SF} of SC families. The best response function f_{SF} gives the optimal allocation θ of the family resources given particular values for p_s, p_k, w_D and w_S.

$$\theta^* = f_{SF}(p_s, p_k, w_D, w_S) \quad (8)$$

Total differentiation of Equation 7 reveals the following comparative statistics of the best response function $f_{MF}(p_s, p_k, w_D)$:

$$\frac{\partial f_{MF}}{\partial p_s} > 0, \qquad \frac{\partial f_{MF}}{\partial p_k} < 0, \qquad \frac{\partial f_{MF}}{\partial w_D} > 0, \qquad \frac{\partial f_{MF}}{\partial w_S} < 0 \quad (9)$$

Elected Official: In contrast to the SC families, the elected official accounts for the effect of borderline security actions on the labor market. With this assumption the utility function for the elected official is given by:

$$u_{EO}(p_s, \theta, \alpha) = v_N(w(Q_N + M\theta p_s)) + \alpha M\theta p_s \quad (10)$$
$$= \tilde{v}_N(Q_N + M\theta p_s)) + \alpha M\theta p_s \quad (11)$$

where $\tilde{v}_N$ is the composite function v_N applied on w (i.e. $\tilde{v}_N = v_N \circ w$).

The optimal level of border security enforcement is determined by the first order condition

$$\frac{\partial u_{EO}}{\partial p_s} = \tilde{v}'_N(Q_N + M\theta p_s))M\theta + \alpha M\theta = 0 \quad (12)$$

which leads to the following best response function for the elected official

$$p_s^* = f_{EO}(\theta, \alpha) = \frac{\tilde{v}'^{-1}_N(-\alpha) - Q_N}{M\theta} \quad (13)$$

3.2. Nash Equilibrium

To obtain Nash Equilbrium levels of $\theta^*(p_k, \alpha, w_S)$ and $p_s^*(p_k, \alpha, w_S)$ we solve the two best response functions:

$$\theta^* = f_{MF}(p_s^*, p_k, \tilde{v}_N(Q_N + M\theta^* p_s^*), w_S) \quad (14)$$
$$p_s^* = \frac{\tilde{v}'^{-1}_N(-\alpha) - Q_N}{M\theta^*} \quad (15)$$

Totally differentiating equations 14 and 15 we get the following comparative statistics for the equilibrium levels of p_s^* and θ^*.

$$\frac{\partial p_s^*}{\partial p_k} > 0, \qquad \frac{\partial p_s^*}{\partial w_S} < 0, \qquad \frac{\partial \theta^*}{\partial p_k} > 0, \qquad \frac{\partial \theta^*}{\partial w_S} < 0 \quad (16)$$

A striking result of our model is seen in the Equation 17: the equilibrium number of immigrant workers is a constant depending only on α, the relative importance of the firms' interest in the elected officials utility function.

$$Q_I^* = Mp_s^*\theta^* = \tilde{v}'^{-1}_N(-\alpha) - Q_N \quad (17)$$

In other words, the equilibrium number of illegal immigrant workers is independent of the variables that only appear in the utility function of the SC families $\{p_k, w_S, \rho\}$. Therefore policies aimed in controlling illegal immigration through these variables are destined to fail, because strategic interaction between the SC families and the elected official cancels any changes introduced in these variables. For example, if the probability of dying p_k increases, perhaps due to an usually hot summer in the border area, in equilibrium the SC families decrease the proportion of their household they send to the destination country. The elected official in the DC, however, relaxes the border enforcement increasing θ, resulting in a constant level of immigration work force. Alternatively, if the wage level in source country

w_S increases, say because of a domestic increase in demand for labor, in equilibrium the SC families will decrease the proportion of their household they send to the DC. The elected official in turn relaxes the border enforcement increasing θ, again resulting in a constant level of immigration work force[7].

These results do not depend on the particular wage preferences of SC families v_S and DC native workers v_N or the fact that the two players choose their actions simultaneously. They are also valid in a Stackelberg formulation of the game where the elected official moves first and commits to a specific value of p_s, followed by the SC families choosing their action after observing the elected officials choice. In the next section we will consider a specific example to illustrate the result and relax the assumption about the deterministic mapping between wages and supply of illegal immigrant workers by introducing uncertainty in the labor market.

4. Uncertainty in the Labor Market

In the previous section we assumed that the elected official can perfectly foresee the effect of the quantity of illegal immigrant workers on the wage level in the country. Here we introduce a simple uncertainty model for the labor market and introduce specific functional forms for the utility function of the DC workers and SC families. We assume that the labor market can be realized in one of the two states with equal probability, one state is a high wage in the DC unskilled market (w_D^H) and the other a low wage (w_D^L). The parameter a determines the wage difference between the two states for a given level of employment.

$$w_D^H(Q_N + Q_I) = (\bar{w} + a) - (Q_N + Q_I) \quad (18)$$
$$w_D^L(Q_N + Q_I) = (\bar{w} - a) - (Q_N + Q_I) \quad (19)$$

Figure 1 illustrates the uncertain demand model for labor. We also assume that both the SC and DC workers' wage preferences are represented by logarithmic utility functions $v_N(w) = v_S(w) = \log w$.

First we note that if both the EO and the SC families choose their strategies after the labor market state is realized, the model of the previous section applies and we obtain the equilibrium outcome in each state by plugging in the corresponding demand function. In the case of uncertainty, the equilibrium is characterized by the strategy pairs $(p_s^*(H), \theta^*(H))$ and $(p_s^*(L), \theta^*(L))$. In general we have the following results:

$$p_s^*(H) > p_s^*(L) \quad (20)$$
$$\theta^*(H) > \theta^*(L) \quad (21)$$

[7] In 1993 the United States Border Patrol launched Operation Blockade to reduce Mexican immigration through El Paso and the following year began Operation Gatekeeper in San Diego. As migrants continued to migrate, but increasingly through the Arizona desert. [?] noted that the death rate per entry attempt, p_k, tripled in the period 1993-1997, while [?] noted that the rate of migration by Mexicans rose. This would be consistent with an increased chance of success in the reaction function after an increase in p_k.

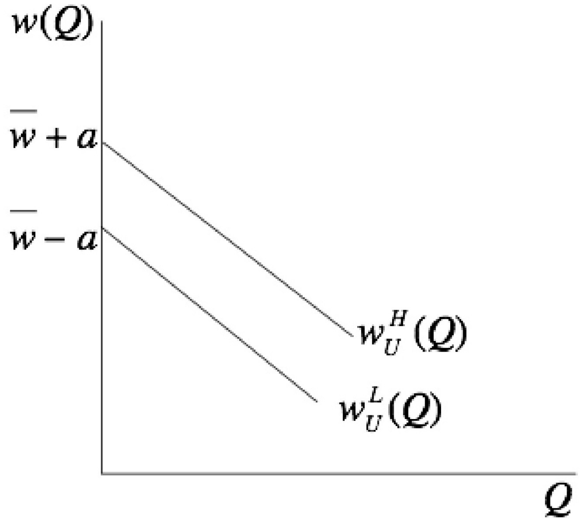

Figure 1: Modeling uncertainty in the labor market

$$Q_I^*(H) = Mp_s^*(H)\theta^*(H) \quad > \quad Q_I^*(L) = Mp_s^*(L)\theta^*(L) \tag{22}$$
$$w_D^*(H) \quad = \quad w_D^*(L) \tag{23}$$

The equality in Equation 23 is a result of our assumption of particular model of the labor market (linear labor demand with varying intercepts between the states). In general we will have weak dependence on the state ($w_D^*(H) \geq w_D^*(L)$). However the main point is that the increase in wages in good times is depressed with increased immigration.

Plainly, favorable market conditions induce SC families to choose to send a higher percentage of their labor force over the border. Perhaps more surprisingly favorable market conditions also induce the elected official to relax the border security because he is not under pressure to keep the wages of the SC workers high. Finally, the overall level of immigration is higher keeping the realized wage level constant.

Figurea 2 and 3 illustrate the main results with a numeric example. First, while the equilibrium levels of p_s and θ depend on exogenous variables, the wages and the level of illegal immigration is constant with respect to the variables that affect *only* the SC families such as probability dying during an attempted border crossing and wage level in SC, p_k and w_S. Second, for a given level of p_k or w_S, favorable labor market conditions in the DC result in higher border crossing effort by SC families and relaxed border security, resulting in higher illegal immigration while keeping the wage level constant.

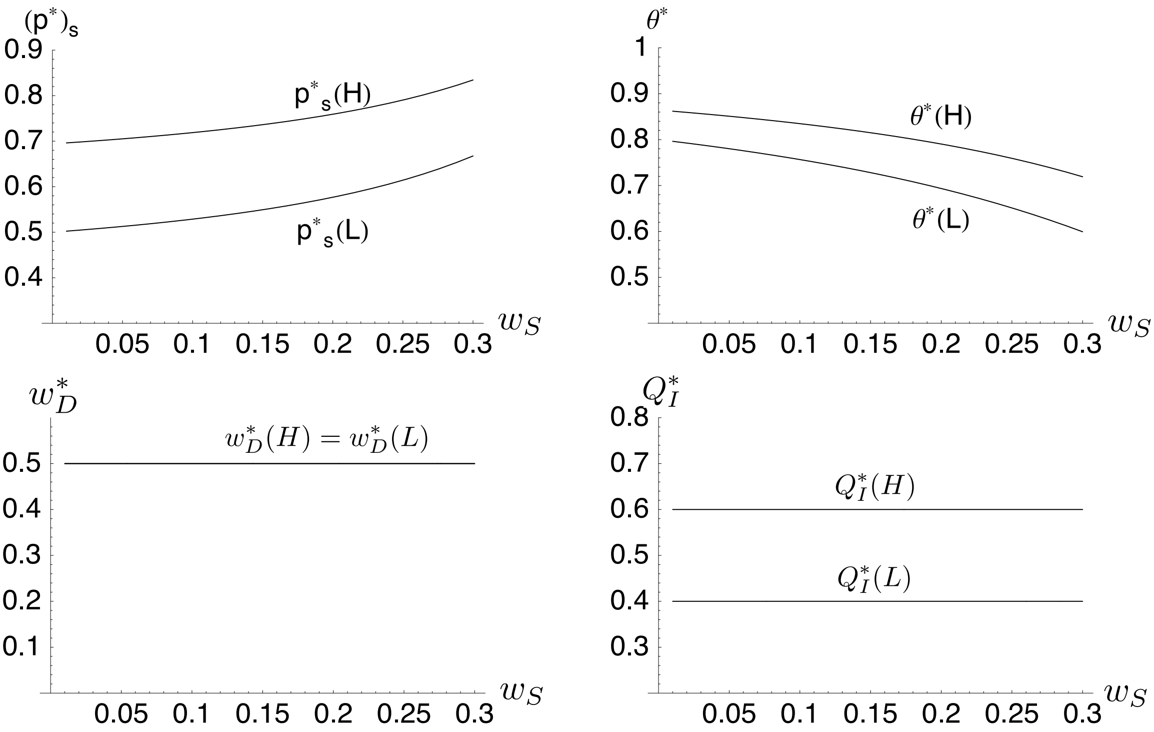

Figure 2: Effect of labor market conditions and the wage level in the source country w_S on immigration model variables ($p_k = 0.1, Q_N = 1, \rho = 0.75, \alpha = 2, M = 1, \bar{w} = 1$)

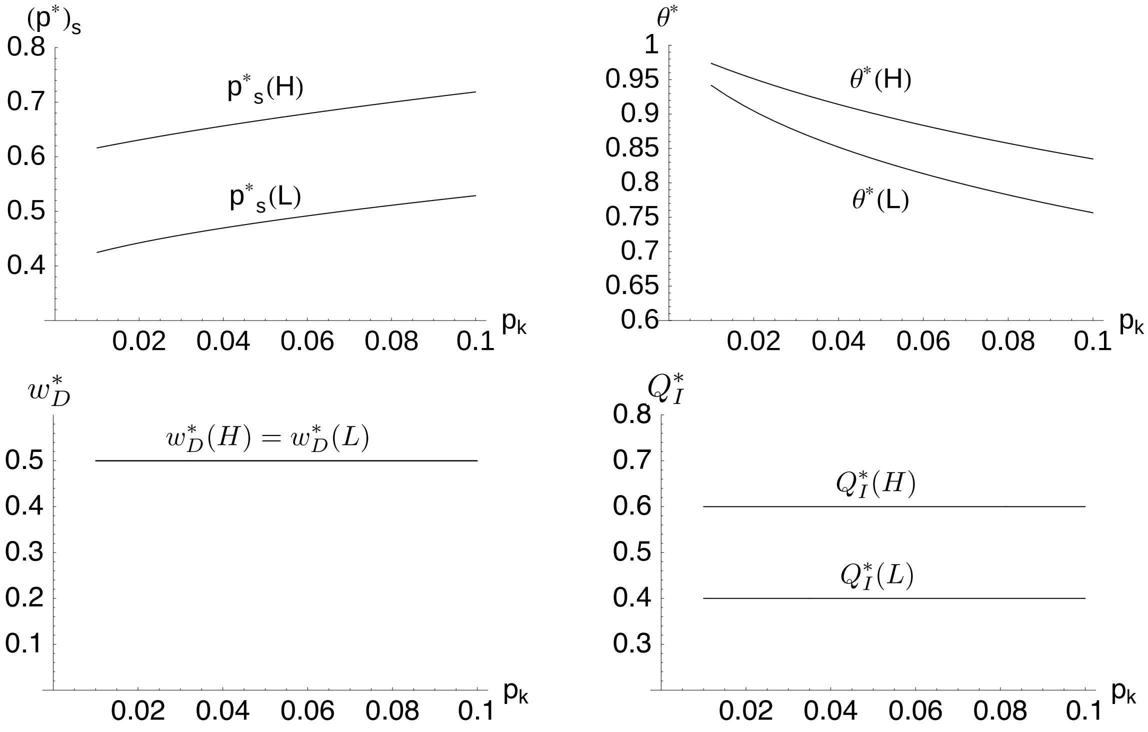

Figure 3: Effect of labor market conditions and probability of being killed p_k on immigration model variables ($w_S = 0.1, Q_N = 1, \rho = 0.75, \alpha = 2, M = 1, \bar{w} = 1$)

We conclude this section with an alternative game theoretic model for strategic interaction under uncertainty. In the following we assume the elected official chooses the border security p_s prior to learning the state of the labor market. This would be the case if the main instruments of border security are of capital nature, building walls, remote sensors etc. As a result of these capital investments the border security decision of the elected official can be viewed as constant over a long time horizon. We assume the SC families will adjust their trial efforts after the realization of the labor market state and will end up in one of the two equilibrium levels of effort θ_h and θ_l. We are looking to find the Bayes-Nash Equilibrium of this immigration game, i.e. the triple $p_s^*, \theta_h^*, \theta_l^*$ where:

$$p_s^* = f_{EO}(\theta_h^*, \theta_l^*, \alpha, M, Q_N, a) \quad (24)$$
$$\theta_h^* = f_{SF}(ps^*, p_k, \rho, w_S, w_h^*) \quad (25)$$
$$\theta_l^* = f_{SF}(ps^*, p_k, \rho, w_S, w_l^*) \quad (26)$$
$$w_h^* = w^H(Q_N + M p_s^* \theta_h^*) \quad (27)$$
$$w_l^* = w^L(Q_N + M p_s^* \theta_l^*) \quad (28)$$

Here the elected official is forced to choose a single effort level that will be effective for both states of the labor demand. Figure 4 shows the results of a numerical example. We observe under this assumptions the level of illegal immigration is affected by the exogenous variables such as the wage level in the source country w_S and the probability of dying or being severely injured while attempting to cross the border p_k. At high demand market state the level of immigration increases with p_k and w_S and at low market state the level of immigration decreases with p_k and w_S. The elected official is trying to choose the value of p_s that will keep the level of immigration at an optimal level balancing the interests of the firms and DC workers. If the elected official has to make its decision before the labor market state is realized, the realized level of immigration will under-or-over shoot this target level depending on which labor market state took place. Furthermore, the amount of over/under shoot is influenced by the exogenous variables such as w_S and p_k amplifying the fluctuations in the labor market. In other words, the elected official sets a target level for the *expected* quantity of immigrants, which results in higher and lower level of immigration than this target level in high and low demand states respectively. We also note that in Figure 4 the gap between the target level and the realized level of illegal immigration increases with w_S and p_k.

5. Conclusion

Illegal immigration is a multifaceted political issue, influenced by the conflicting interests of politicians, firms, workers in the destination country and potential immigrants in the source country. In this paper we considered a stylized model based on a game theoretic model with an aim to emphasize the strategic interaction between the different forces affecting the issue. In particular, we have shown that exogenous changes affecting only some of the players

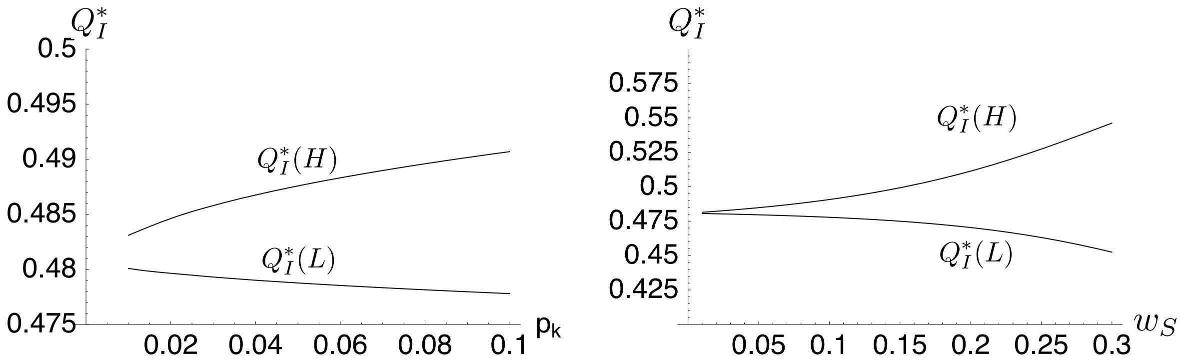

Figure 4: Effect of wage rate in the source country w_S and probability of being killed p_k on the equilibrium number of immigrant workers ($Q_N = 1, \rho = 0.75, \alpha = 2, M = 1, \bar{w} = 1$)

might have effects of an unexpectedly small magnitude because strategic interaction between the players will tend to cancel any unilateral changes. For example, if the wage rate in the source country were to rise, the net result would be the sending of fewer family members to the destination country. Perhaps surprisingly, however, the probability of a successful crossing would also increase as the elected official in the destination country would relax border enforcement with the reduction in attempted crossing. We have also shown that illegal immigration is actually beneficial in smoothing the fluctuations in the labor market as long as both parties are allowed to react to changes in the labor market conditions. The actions of the various parties result in stable wages, with the varying number of the immigrants in the labor force stabilizing demand shocks.

The results of the model certainly have implications for events that are currently unfolding. In the recent years, in US, private citizens started patrolling the border increasing the risk on illegal workers attempting to migrate. Our results indicate that the net results will be a higher probability of success among a smaller group of people attempting to migrate. Given that the probability of a successful crossing rises as does the chances of death, the result will be a fall in the share returning after an unsuccessful attempt at crossing the border. The model may be applied to other changes as well.