Please perform all calculations without the use of software aides such as Mathematica. You may use a calculator, however.

1. (4 points) Suppose ten students each measure the diameter of a steel ball with a micrometer caliper. For a variety of reasons we do not expect all of the measurements to be identical. The sources of deviation could include:
   - The steel ball may not be perfectly round.
   - The ball may not be centered between the jaws of the caliper when the diameter is measured.
   - The temperature of the ball may increase with time as the ball is handled and hence its diameter may change slightly through thermal expansion.
   - There may be varying amounts of corrosion on the ball.

Which of the above sources of deviation contribute to systematic uncertainty? Which contribute to random uncertainty? (Note: It may be possible to argue that some of the situations contribute to both types of uncertainty, so it is important that you explain your reasoning.)

2. The mass of a sample of iron was measured in the laboratory. The manufacturer claims that the mass of the sample is 10.00 g, within 0.4%.
   (a) (1 point) Determine the absolute uncertainty in the mass according to the manufacturer.
   (b) (3 points) Suppose you measured the mass of the sample and obtained the value $9.918 \pm 0.023$ g. Discuss whether your result agrees with the manufacturer’s claim.

3. (4 points) In an experiment with an air track, an experimenter wishes to determine the average speed of an air track cart between two photogate timers. The distance $\Delta x$ between the photogates was measured to be $\Delta x = 1.000 \pm 0.003$ m, and the time of travel $\Delta t$ between these two points was measured to be $\Delta t = 2.3 \pm 0.1$ s. Calculate the average speed $s = \Delta x / \Delta t$, the absolute uncertainty $\sigma_s$, and the fractional uncertainty $\varepsilon_s$ in the average speed given these data.

4. (a) (2 points) To measure the density of a rectangular object, an experimenter measures the object’s volume and mass. The volume is given by the formula $V = LWH$, where $L$ is the length, $W$ is the width, and $H$ is the height. The density $\rho$ is given by $\rho = m / V$, where $m$ is the object’s mass. If the measurement of the mass is uncertain by 3%, and each of $L$, $W$, and $H$ is uncertain by 4%, what is the uncertainty, in percent, of the density $\rho$?
   (b) (3 points) The experimenter conducts the same density measurement with a second sample that is spherical in shape. The mass is again uncertain by 3%. The
diameter \( d \) of the sphere is measured to a precision of 4%. The volume \( V \) of a sphere is given by the formula

\[
V = \frac{4\pi}{3} \left( \frac{d}{2} \right)^3.
\]

What is the fractional uncertainty of the density \( \rho \) in this case? What is the underlying reason why the uncertainty in this case is different than in the case of a rectangular object?

5. (5 points) Your data are a set of measurements of the length of a sheet of paper, made with a 30-cm rule. They are as follows (each with an uncertainty of ± 0.01 cm):

\[
\begin{align*}
L_1 &= 27.94 \text{ cm} \quad L_2 = 27.96 \text{ cm} \quad L_3 = 27.99 \text{ cm} \quad L_4 = 27.97 \text{ cm} \\
L_5 &= 28.00 \text{ cm} \quad L_6 = 27.93 \text{ cm} \quad L_7 = 27.96 \text{ cm} \quad L_8 = 27.98 \text{ cm}
\end{align*}
\]

Calculate:

(a) the mean value
(b) the standard deviation \( \sigma \)
(c) the standard deviation of the mean \( \alpha \)
(d) the result to be reported
(e) the weighted average if all but \( L_6 \)'s measurement uncertainty were changed to ± 0.10 cm

6. (5 points) Suppose that three measured lengths and their estimated errors are:

\[
\begin{align*}
L_1 \pm \Delta L_1 &= 23.5 \pm 0.1 \text{ cm} \quad L_2 \pm \Delta L_2 = 17.8 \pm 0.2 \text{ cm} \quad L_3 \pm \Delta L_3 = 93.9 \pm 0.2 \text{ cm}
\end{align*}
\]

If the quantity to be calculated, \( L \), is defined to be \( L = L_1 + 2L_2 - L_3 \), then determine the reported value of \( L \) with its error.

7. (5 points) Suppose two time intervals and their uncertainties are \( t_1 \pm \Delta t_1 = 0.743 \pm 0.005 \text{ s} \) and \( t_2 \pm \Delta t_2 = 0.384 \pm 0.005 \text{ s} \). If the total time \( t \) is defined to be \( t = 2t_1 + 5t_2 \), then determine the reported value of \( t \) with its uncertainty.

8. (5 points) Newton’s Law of Gravitation states that \( F = \frac{Gm_1m_2}{r^2} \). Suppose

\[
\begin{align*}
m_1 \pm \Delta m_1 &= 19.7 \pm 0.2 \text{ kg} \\
m_2 \pm \Delta m_2 &= 9.4 \pm 0.2 \text{ kg} \\
r \pm \Delta r &= 0.641 \pm 0.009 \text{ m} \\
G &= 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}
\end{align*}
\]

Determine the reported value of \( F \) with its uncertainty.
9. (5 points) Snell’s law of refraction states that \( n_2 \sin \theta_2 = n_1 \sin \theta_1 \). Suppose

\[
\begin{align*}
    n_1 & = 1.000 \\
    \theta_1 & = (61 \pm 2)^\circ \\
    \theta_2 & = (36 \pm 1)^\circ 
\end{align*}
\]

Determine the reported value of \( n_2 \) with its uncertainty.

10. (8 points) The period of underdamped harmonic motion is given by

\[
T = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}},
\]

where

\[
\begin{align*}
    k & = 0.11 \pm 0.01 \text{ N/m} \\
    m & = 0.500 \pm 0.005 \text{ kg} \\
    b & = 0.062 \pm 0.008 \text{ kg/s} 
\end{align*}
\]

Determine the reported value of \( T \) with its uncertainty, along with the fractional uncertainty in \( T \).