**Homework III - solutions**

1. **Saving** The effect of a government tax increase of $100 billion on (a) public saving, (b) private saving, and (c) national saving can be analyzed by using the following relationships:

   \[
   \text{National Saving} = [\text{Private Saving}] + [\text{Public Saving}]
   = [Y - T - C(Y - T)] + [T - G]
   = Y - C(Y - T) - G
   \]

   a. **Public Saving.** The tax increase causes a 1-for-1 increase in public saving. \( T \) increases by $100 billion and, therefore, public saving increases by $100 billion.

   b. **Private Saving.** The increase in taxes decreases disposable income, \( Y - T \), by $100 billion. Since the marginal propensity to consume (\( MPC \)) is 0.6, consumption falls by \( 0.6 \times 100 \) billion, or $60 billion. Hence,

   \[
   \Delta \text{Private Saving} = -100b - 0.6(-100b) = -40b
   \]

   Private saving falls $40 billion.

   c. **National Saving.** Because national saving is the sum of private and public saving, we can conclude that the $100 billion tax increase leads to a $60 billion increase in national saving. Another way to see this is by using the third equation for national saving expressed above, that national saving equals \( Y - C(Y - T) - G \). The $100 billion tax increase reduces disposable income and causes consumption to fall by $60 billion. Since neither G nor Y changes, national saving thus rises by $60 billion.

   d. **Investment.** To determine the effect of the tax increase on investment, recall the national accounts identity:

   \[
   Y = C(Y - T) + I(r) + G
   \]

   Rearranging, we find

   \[
   Y - C(Y - T) - G = I(r)
   \]

   The left-hand side of this equation is national saving, so the equation just says the national saving equals investment. Since national saving increases by $60 billion, investment must also increase by $60 billion.
How does this increase in investment take place? We know that investment depends on the real interest rate. For investment to rise, the real interest rate must fall. Figure below graphs saving and investment as a function of the real interest rate.

The tax increase causes national saving to rise, so the supply curve for loanable funds shifts to the right. The equilibrium real interest rate falls, and investment rises.

2. **Demand for Investment** If there is an increase the demand for investment, then:

(a) the investment curve shifts up (*remember that the investment curve represents demand for investment*)

(b) In words:
   i. the real interest rate increases
   ii. national saving is unchanged
   iii. amount of investment is unchanged
   iv. consumption is unchanged
   v. output is unchanged ⇒ fixed because it is determined by the factors of production
3. **Capital Destruction** First we need to find expressions for marginal productivities of capital and labor. Since the production function is $Y = AK^\alpha L^{1-\alpha}$,

$$MPK = \frac{\alpha Y}{K} = \alpha AK^{\alpha-1}L^{1-\alpha} = \alpha A \frac{L^{1-\alpha}}{K^{1-\alpha}}$$

$$MPK = \alpha A \left( \frac{L}{K} \right)^{1-\alpha}$$

$$MPL = \frac{(1-\alpha)Y}{L} = (1-\alpha)AK^{\alpha}L^{-\alpha} = (1-\alpha)A \frac{K^{\alpha}}{L^{\alpha}}$$

$$MPL = (1-\alpha)A \left( \frac{K}{L} \right)^{\alpha}$$

(a) Since the quantity of $K$ decreases, output will decrease (less input $\Rightarrow$ less output). However, output decreases by less than 50% because of diminishing returns to capital.

(b) The real wage decreases because the average productivity of labor decreases ($Y/L$ decreases, as $Y$ decreases and $L$ is constant), so the $MPL$, which equals $(1-\alpha)Y/L$, decreases ($K$ is in the numerator of the $MPL$ and smaller $K$ means smaller $MPL$, which equals the real wage rate).

(c) The real rental price of capital increases because the average productivity of capital increases ($Y/K$ increases, as $Y$ decreases proportionally less than $K$ decreases), so the $MPK$, which equals $\alpha Y/K$, increases ($K$ is in the denominator of the $MPK$ and smaller $K$ means larger $MPK$, which equals the real rental rate of capital).

(d) Capital's share of income is unchanged since it depends only on the parameter ($\alpha$) from the production function, which does not change.

**Multiple Choice**

4. In the classical model with fixed income, if households want to save more than firms want to invest, then:

   (b) the interest rate falls.

5. In a classical model with fixed factors of production and flexible prices, the amount of consumption spending depends on ________, the amount of investment spending depends on ________, and the amount of government spending is determined ________.

   (a) disposable income; the interest rate; exogenously

6. When a pizza maker lists the price of a pizza as $10, this is an example of using money as a:
(c) unit of account

7. To increase the money supply, the Federal Reserve:

(b) buys government bonds